ON THE LIFT FORCE AT HYPERSONIC SPEEDS

(O POD"EMNOI SILE PRI GIPERZVUKOVYKH SKOROSTIAKH)

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V.S. GALKIN and A.A.GLADKOV
(Moscow)

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Attention is called to the fact that the lift force of a broad class of bodies (wedges, cones, etc.) at hypersonic speeds and arbitrary values of the Knudsen number can be negative for an arbitrary angle of attack α in the interval $0 < \alpha \leqslant \pi/2$. Let us take for consideration a very simple case: flow past a wedge with semi-vertex angle δ in free-molecular flow, with velocity $V \gg c$, i.e. $S = V/c \gg 1$, where c is the most probable thermal velocity of the molecules impinging on the body. Thus the thermal velocities of the impinging molecules may be neglected, and it may be assumed that they move parallel to one another with uniform velocity V. Then with diffuse reflection all the impulses of the impinging molecules are spent on the drag force of the body, whilst the lift force is created by the impulses of the reflected molecules, the force of reaction to which is directed along the inward normal to the surface element of the body.

Suppose that the drag force is directed along the axis of x, parallel to V, and the lift force along the axis of y, obtained by rotating the vector V counter-clockwise through a right-angle. The angle of attack is the angle between the x-axis and the axis of symmetry of the wedge.

The lift force on the lower face of the wedge is

$$Y_+ \approx \sin\ 2\ (\delta + \alpha)$$

The lift force on the upper face of the wedge is

$$Y_- \approx \sin 2(\alpha - \delta) \text{ when } \delta > \alpha, \quad Y_- = 0 \text{ when } \delta \leqslant \alpha$$

In other words, the lift force on the wedge is

$$Y \approx \sin\ 2\alpha \cos\ 2\delta \text{ when } \delta > \alpha, \quad Y \approx \sin\ 2\ (\delta + \alpha) \text{ when } \delta \leqslant \alpha$$

Hence it follows that $Y \leqslant 0$ when $\delta \geqslant \pi/4$ for any angle of attack, and $Y \leqslant 0$ when $\delta < \pi/4$ and $\alpha \geqslant \pi/2 - \delta$ $(\alpha \geqslant \delta)$.

In the latter case the aerodynamic forces act only on the lower face. If $a \geqslant \pi/2 - \delta$, then its angle of attack is greater than or equal to $\pi/2$, and the projection upon the y-axis of the force of reaction to the momentum of the reflected particles is negative or equal to zero. It can be shown that in the case of the circular cone $Y \leqslant 0$ for any angle of $\leqslant$ attack if $\sin \delta \geqslant 1/\sqrt{3}$. This result is easily demonstrated. If the angle of attack of a wedge or cone $a \approx \pi/2$, then always $Y < 0$.

Let us consider the case of an arbitrary angle of attack. We shall increase δ up to $\delta = \pi/2$. The force of reaction to the momentum of the reflected molecules is directed along the inward normal to the flat plate thus obtained: moreover, the angle between this normal and the y-axis is greater than $\pi/2$. Consequently, the projection on the y-axis of the force of reaction to the momentum of the reflected molecules is negative. From consideration of continuity it follows that when $\delta < \pi/2$ there is a certain range of values of δ such that $Y < 0$.

Let us study now the case $S \ll 1$. The stream of impulses on all the elements of the surface is approximately uniform, and the force acting on an element of the surface is directed approximately perpendicular to it. Since the projection on the y-axis of the unit vector normal to the upper surface of the wedge is greater than the corresponding projection in the case of the lower surface, then for any angle of attack of the infinite wedge the pressure on the upper surface of the wedge is greater than that on the lower, i.e. $Y < 0$.

Summarising the results obtained, we arrive at the conclusion that for any S there are values of δ greater than a certain $\delta = \phi$, for which the lift force of an infinite wedge is negative. With $S > 2$, when the base pressure can be neglected, this conclusion carries over to the real case of a wedge of finite length.

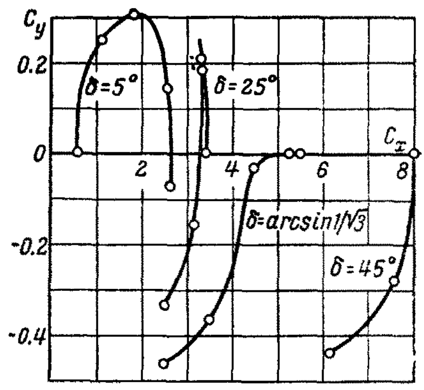
Obviously, this conclusion is true for a broad class of bodies. Exceptions are bodies of the type of cylinders and flat plates at an angle of attack, for which always $Y \geqslant 0$.

In the general case the values of those geometrical parameters of bodies for which $Y < 0$ depend on the shape of the body, the magnitudes of S and the accommodation and reflection coefficients.

Depending on the shape of the body and the values of the coefficients just mentioned, the drag of bodies of the class under consideration may either increase along with the angle of attack, or may decrease.

In the case of hypersonic velocities the conclusions we have obtained are especially true for large values of the ratio of the temperature of reflected molecules T_r to the temperature of the unperturbed stream T.

In the figure we show the polars of cones for the case when $T_r/T = S^2/5$ and $S \gg 1$. The small circles denote corresponding values of the aerodynamic coefficients for $\alpha = 0, 20, 40, 70, 90°$. Reflection is assumed to be purely diffusive.



It is easy to show that the conclusions obtained for an important class of bodies, the lift force of which is negative for any angle of attack in the range $0 < \alpha \leqslant \pi/2$, are valid also for the case of a body in a hypersonic stream of a continuous medium, when the distribution of pressure on the body can be calculated by Newtonian theory. In fact, let us consider the very simple case of flow of a hypersonic inviscid stream past a wedge. Then

$$Y_+ \approx \sin(\delta + \alpha)\sin 2(\delta + \alpha), \quad Y_- \approx \sin(\alpha - \delta)\sin 2(\delta - \alpha) \quad \text{when } \delta > \alpha$$

$$Y_- = 0 \ \text{ when } \delta \leqslant \alpha, \ Y < 0, \ \text{ if } \ \alpha \approx \pi/2$$

$$Y \leqslant 0 \text{ when } \delta \leqslant \alpha, \ \text{ if } \ \alpha + \delta \geqslant \pi/2$$

When the angle of attack is small, then from the formulas derived above it follows that

$$Y \approx \alpha \sin\delta\,(2 - 3\sin^2\delta) \quad \text{when } \delta > \alpha$$

Using these estimates, it can be shown that $Y \leqslant 0$ in the interval $0 \leqslant \alpha \leqslant \pi/2$, when $\sin\delta \geqslant \sqrt{2/3}$. It is obvious that the conclusion concerning the negative lift force of the class of bodies under consideration is valid for specular reflection in free-molecular flow and in general for any type of reflection.

The results obtained enable us to draw the conclusion that for any values of the Knudsen number the lift force of a broad class of bodies (wedges, cones, etc.) in a hypersonic stream is negative for any values of the angle of attack in the interval $0 < \alpha \leqslant \pi/2$.

Translated by A.H.A.